Exponents and Powers

An expression that consists of a repeated power of multiplication of the same factor is called as Power/Exponent/Indices.

The **power** of a number indicates the number of times it must be multiplied. It is written in the form a^b. Where '**b**' indicates the number of times 'a' needs to be multiplied to get our result. Here 'a' is called the **base** and '**b**' is called the **exponent**.

For example: Consider 9^3. Here the exponent '3' indicates that base '9' needs to be multiplied three times that is $9^3 = 9x9x9 = 729$.

Consider an example like 5^2, the number 5 is called the base, whereas 2 is the power/indices/exponent of the expression.
The value of the expression is derived by multiplying the base as many time as the number of power. In the example above, the power is 2, thus the value becomes 5×5=25.

NOTE- The number a^n is read as 'a raised to the power of n' or simply as 'nth power of a'. The notation an is called the exponential notation or power notation.

Types of Exponents

Exponents can be divided into four types based on the number in the power. They are:

1. Positive exponent
2. Negative exponent
3. Zero exponent
4. Rational exponent (not to be studied in class VIII)

- **Positive exponents** can be simplified just by multiplying the base to itself the number of times indicated by the exponent/power.
- A **negative exponent** can be simplified by placing 1 in the numerator and the base along with the exponent in the denominator of a fraction.
- **Zero exponents** Any expression with the exponent as 0 is equal to 1 and no need to consider the base value during simplification.

Powers with Negative Exponents

A **negative exponent** in power for any non-integer is basically a **reciprocal** of the power.

In simple terms, for a non-zero integer **a** with an exponent **-b**, ie, $a^{-b} = \frac{1}{a^b}$

Expanding a Rational Number Using Powers

A given **rational number** can be expressed in expanded form with the help of **exponents**. Consider a number 1204.65. When expanded the number can be written like :
$1204.65=1000+200+4+0.6+0.05=(1\times10^3)+(2\times10^2)+(0\times10^1)+(4\times10^{-1})+(5\times10^{-2})$

Laws of Exponents

If a, b are non-zero integers and m, n are any integers, then

(i) $a^m \times a^n = a^{m+n}$

(ii) $\dfrac{a^m}{a^n} = a^{m-n}$

(iii) $(a^m)^n = a^{mn}$

(iv) $a^m \times b^m = (ab)^m$

(v) $\dfrac{a^m}{b^m} = \left(\dfrac{a}{b}\right)^m$

(vi) $a^0 = 1$

(vii) $\left(\dfrac{a^{-m}}{b^{-n}}\right) = \dfrac{b^n}{a^m}$

(viii) $\left(\dfrac{a}{b}\right)^{-m} = \left(\dfrac{b}{a}\right)^m$

Remember

- $a^n = 1 \Rightarrow n = 0$
- $1^n = 1$ where n is any integer.
- $(-1)^n = 1$ where n is any even integer.
- $(-1)^n = -1$ where n is any odd integer.

Uses of Exponents

Inter Conversion between Standard and Normal Forms

Very **large numbers** or very **small numbers** can be represented in the **standard form** with the help of **exponents**.

If it is a very large number like 150,000,000,000, then we need to move the **decimal place towards the left**. And when we do so the exponent will be **positive.**

150000000000.

11 10 9 8 7 6 5 4 3 2 1

Since the **decimal** is moved **11 places** till it is placed between 1 and 5, our standard form representation of the large number will be 1.5×10^{11}

If it is a very small number like 0.000007, we need to move the **decimal places to the right** in-order to represent the number in its standard form. When being shifted to the right, the exponent will be **negative.**

0.000007

1 2 3 4 5 6

In this case, the decimal place is moved **6** places up until till it is placed after digit 7. Therefore our standard form representation will be

7×10^{-6}

The exponents are also useful when converting the number from it's standard form to it's natural form.

Comparison of Quantities Using Exponents

In-order to **compare two large or small quantities**, we convert them to their standard exponential form and divide them.

For example : To compare the diameter of the earth and that of the sun.

Diameter of the Earth = 1.2756×10^6m

Diameter of the Sun = 1.4×10^9m

Diameter of Sun = 1.4×10^9m
Diameter of the Earth =1.2756×10^7m

=109

So the diameter of the Sun is 109 times that of the Earth!

While calculating the total or the difference between two quantities, we must first ensure that the exponents of both the quantities are the same.

6·8·21

Classwork
Exponents

Q1) $\frac{1}{2} = 2^{-1}$ → power
↓
base

2) $3^{-1} = \frac{1}{3}$

3) $4^{-1} = \frac{1}{4}$

4) $10^{-2} = \frac{1}{10^2}$

5) $3^0 = 1$

6) $3^1 = 3$

7) $3^2 = 9$

8) $3^3 = 27$

Q2

Q2 Find the reciprocal of

	No.	Value	M. I.
①	2^{-4}	$\frac{1}{24}$	24
②	10^{-5}	$1/105$	105
③	7^{-2}	$1/7^2$	7^2
④	5^{-3}	$1/5^3$	5^3
⑤	10^{-100}	$1/10^{100}$	10^{100}

Q① Expand

1 $1025.63 = 1\times10^3 + 0\times10^2 + 2\times10^1 + 5\times10^0 + 6\times10^{-1} + 3\times10^{-2}$

2 $1256.249 = 1\times10^3 + 2\times10^2 + 5\times10^1 + 6\times10^0 + 2\times10^{-1} + 4\times10^{-2} + 9\times10^{-3}$